

## **Alerting Students to the Dangers of Misstating Equations in the Reciprocal Method of Cost Allocation**

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### **Overview**

Cost accounting courses usually cover the direct, step-down and reciprocal (or simultaneous) techniques for cost allocation. In today's technologically-oriented environment, it is often too tempting to leave the most difficult calculations to the computer. While this computer-default mode can pose little risk for many business calculations, this is not true for calculations involving the reciprocal cost allocation method. Students need to be alerted about the care that should be taken to ensure that the correct cost relationships are captured when the cost equations are specified. Once correctly specified, the equations can then be solved by computer, if desired. In this paper, examples are presented that demonstrate to students just how easy it is to misstate the cost equations and how data presentation techniques can be used to provide double-checks to ensure accurate answers when using the reciprocal method. In addition, an Excel iterative approach to the reciprocal method is demonstrated.

### **A Basic Example to Demonstrate the Dangers of Misstatement of the Cost Equations**

The following example includes three production and two service departments. It actually could be used as either an in-class or an exam question since it can be solved by using either addition and subtraction or substitution, with no computer needed.

Reciprocal Company's two service departments serve not only the three production departments, but also one another. The relationships among the five departments can be expressed as follows:

Service Departments	Percentage of Services Consumed by Departments					Service Costs To Be Distributed
	Producing			Service		
	1	2	3	A	B	
A	25%	15%	40%			\$32,314
B	35%	20%	15%	30%	20%	\$24,200

Required: Determine the allocations to be made to each department.

To solve this problem, the first (and most crucial) step is to correctly specify the cost equations for service departments A and B. Students sometimes look at the chart and misstate the equations as follows:

$$A = \$32,314 + .2B$$

$$B = \$24,200 + .3A$$

Looking at the way the data is presented in the chart, it can be seen why the students might misstate the equations. The focus in the misstated equations is on where A's and B's costs are going – not on what costs are coming into A and B. When will students discover their misstatement mistake? Unfortunately, they will not discover it by solving the equations or distributing the costs. For example, the equations will generate multipliers, as follows:

$$A = \$32,314 + .2 [\$24,200 + .3A]$$

$$A = \$32,314 + \$4,840 + .06A$$

$$.94A = \$37,154$$

$$A = \underline{\underline{\$39,526}}$$

$$B = \$24,200 + .3A$$

$$B = \$24,200 + .3 (\$39,526)$$

$$B = \$24,200 + \$11,858$$

$$B = \underline{\underline{\$36,058}}$$

These multipliers can then be used to distribute the costs in the percentages indicated in the chart. Nothing alerts the students that, even though the equations solved above have no mathematical errors, the answers are wrong because the original equations were misstated. One solution to this dilemma can be found in the way students are encouraged to present their results.

Having students develop (or fill in on an exam) a schedule similar to the following will reveal a mistake in the specified cost equations, if one is present.

Required: Fill in the following schedule of the service costs allocated to each department. (Assume that costs in departments 1, 2 and 3 were \$70,000, \$50,000 and \$20,000 respectively before allocation.)

Total Costs Before Allocation	A	B	1	2	3
	\$32,314	\$24,200	\$70,000	\$50,000	\$20,000
Allocation of A	- _____	+ _____	+ _____	+ _____	+ _____
Allocation of B	+ _____	- _____	+ _____	+ _____	+ _____
<hr/>					
Total Costs After Allocation	-0-	-0-	=====	=====	=====

Upon completion, students will discover the following:

Total Costs Before Allocation	A	B	1	2	3
	\$32,314	\$24,200	\$70,000	\$50,000	\$20,000
Allocation of A	- <u>39,526</u>	+ <u>7,905</u>	+ <u>9,882</u>	+ <u>5,929</u>	+ <u>15,810</u>
Allocation of B	+ <u>10,817</u>	- <u>36,058</u>	+ <u>12,620</u>	+ <u>7,212</u>	+ <u>5,409</u>
<hr/>					
Total Costs After Allocation	\$ 3,605 but should be -0-	(\$3,953) but should be -0-	<u>\$92,502</u>	<u>\$63,141</u>	<u>\$41,219</u>

Since the costs in Departments A and B are not totally allocated, this should alert students to the need to return to the original problem and double-check for the possibility of a misstatement of the cost equations. Upon reexamination, students should see that the total cost equation for Department A is \$32,314 plus what is coming into Department A from Department B, or .3B. Similarly, the total cost equation for Department B is \$24,200 plus what is coming into Department B from Department A, or .2A. Consequently, the correctly specified cost equations are as follows:

$$A = \$32,314 + .3B$$

$$B = \$24,200 + .2A$$

The equations can be solved using substitution, as follows:

$$A = \$32,314 + .3 [\$24,200 + .2A]$$

$$A = \$32,314 + \$7,260 + .06A$$

$$.94A = \$39,574$$

$$A = \underline{\$42,100}$$

$$B = \$24,200 + .2A$$

$$B = \$24,200 + .2 (\$42,100)$$

$$B = \$24,200 + \$8,420$$

$$B = \underline{\$32,620}$$

When these multipliers are used to distribute the costs in the chart, the following results occur:

Total Costs Before Allocation	A	B	1	2	3
	\$32,314	\$24,200	\$70,000	\$50,000	\$20,000
Allocation of A	- <u>42,100</u>	+ <u>8,420</u>	+ <u>10,525</u>	+ <u>6,315</u>	+ <u>16,840</u>
Allocation of B	+ <u>9,786</u>	- <u>32,620</u>	+ <u>11,417</u>	+ <u>6,524</u>	+ <u>4,893</u>
Total Costs After Allocation	-0-	-0-	<u>\$91,942</u>	<u>\$62,839</u>	<u>\$41,733</u>

Accordingly, the distribution chart above shows students that the multipliers used were correct since the goal of completely allocating the service department costs to the production departments (with -0- left in Departments A and B) has been achieved.

### A More Complex Example

The following is a more advanced problem involving three production departments and three service departments (and consequently three simultaneous equations). This is a good problem for more advanced and/or graduate cost students to first solve by hand and then solve using the computer to compare the results. This problem presents a more complex dilemma of correctly specifying and solving each of three service department cost equations.

Simultaneous Company's three service departments serve not only the three production departments, but also one another. The relationships among the six departments can be expressed as follows:

Service Departments	Percentage of Services Consumed by Departments						Service Costs To Be Distributed
	Producing			Service			
	1	2	3	A	B	C	
A	20%	20%	30%		20%	10%	\$ 50,000
B	10%	30%	20%	10%		30%	\$170,000
C	30%	10%	30%	20%	10%		\$120,000

The first step is to correctly specify the cost equations for each of the service departments: A, B and C. The accurately-specified equations are as follows:

$$A = \$50,000 + .1B + .2C$$

$$B = \$170,000 + .2A + .1C$$

$$C = \$120,000 + .1A + .3B$$

Three unknowns are most quickly solved by using the computer. In fact, as previously mentioned, students might work this problem both by hand and by using the computer and then compare the results. Such a comparison shows the rather laborious manual work that the computer does quickly. However, the computer uses the student-specified equations. So, wrongly-specified equations equal quickly-generated, but wrong computer answers. The following is one way to solve the equations by hand, first by using addition and subtraction to the point of two unknowns, and then by using substitution to finish solving for the unknowns. (Here, only the correct equations are used.)

Rearranging the three equations:

$$\begin{array}{r} A - .1B - .2C = \$ 50,000 \\ -2A + B - .1C = \$170,000 \\ -.1A - .3B + C = \$120,000 \end{array}$$

Multiply by -2 =  $\frac{.4A - 2.0B + .2C = - 340,000}{1.4A - 2.1B = -\$290,000}$       Multiply by 10 =  $\frac{-.1A - .3B + C = \$ 120,000}{-2.0A + 10.0B - C = 1,700,000}$   
 $\frac{-2.1A + 9.7B = \$1,820,000}{}$

Multiply by 9.7 =  $\frac{13.58A - 20.37B = -\$2,813,000}{-4.41A + 20.37B = 3,822,000}$       Multiply by 2.1 =  $\frac{9.17A = \$1,009,000}{A = \underline{\$ 110,033}}$

Substituting to find B:

$$\begin{array}{r} 1.4A - 2.1B = -\$290,000 \\ 1.4(\$110,033) - 2.1B = -\$290,000 \\ \$154,046 - 2.1B = -\$290,000 \\ -2.1B = -\$444,046 \\ B = \underline{\$211,450} \end{array}$$

Substituting to find C:

$$\begin{array}{r} C = \$120,000 + .1A + .3B \\ C = \$120,000 + .1(\$110,033) + .3(\$211,450) \\ C = \$120,000 + \$11,003 + \$63,435 \\ C = \underline{\$194,438} \end{array}$$

When these multipliers are used to distribute the costs in a distribution chart, the following results occur: (Assume that costs in departments 1, 2 and 3 were \$70,000, \$130,000 and \$210,000 respectively before allocation.)

Total Costs Before Allocation	A	B	C	1	2	3
	\$ 50,000	\$170,000	\$120,000	\$ 70,000	\$130,000	\$210,000
Allocation of A	- <u>110,033</u>	+ <u>22,006</u>	+ <u>11,003</u>	+ <u>22,007</u>	+ <u>22,007</u>	+ <u>33,010</u>
Allocation of B	+ <u>21,145</u>	- <u>211,450</u>	+ <u>63,435</u>	+ <u>21,145</u>	+ <u>63,435</u>	+ <u>42,290</u>
Allocation of C	+ <u>38,888</u>	+ <u>19,444</u>	- <u>194,438</u>	+ <u>58,331</u>	+ <u>19,444</u>	+ <u>58,331</u>
Total Costs After Allocation	-0-	-0-	-0-	<u>\$171,483</u>	<u>\$234,886</u>	<u>\$343,631</u>

As in the previous problem, the distribution chart above shows students that the multipliers used were correct since the goal of completely allocating the service department costs to the production departments (with -0- left in Departments A, B and C) has been achieved.

An alternative way to solve the multiple-service department allocation

problem is to use an iterative approach with Excel. Once the appropriate formulas are input into the spreadsheet, solving the problem is a matter of copying and pasting new iterations until all support department costs are allocated. The Excel spreadsheet is presented in Appendix A

## **Summary and Conclusions**

Both examples in this paper highlight the ease with which cost equations developed to solve cost allocation problems using the reciprocal method can be misstated. If this first step of specifying the equations is faulty, the time and effort spent solving the problem, either by hand or with a computer is fruitless and will give sometimes hard-to-detect wrong answers. However, double-checking the answers by using distribution tables like those shown in this paper or by using an Excel spreadsheet can prove invaluable in helping students ensure that the time and effort spent in calculating the allocation of costs using the reciprocal method reliably produces accurate results.

### Appendix A: Excel Spreadsheet

Service Departments	Percentage of Support Services Consumed by Other Departments					
	Service Departments			Production Departments		
	A	B	C	1	2	3
A	0%	20%	10%	20%	20%	30%
B	10%	0%	30%	10%	30%	20%
C	20%	10%	0%	30%	10%	30%

Iterations	Service Departments			Production Departments			
	A	B	C	1	2	3	
	50,000.00	170,000.00	120,000.00	70,000.00	130,000.00	210,000.00	750,000.00
1 - A	(50,000.00)	10,000.00	5,000.00	10,000.00	10,000.00	15,000.00	-
1 - B	17,000.00	(170,000.00)	51,000.00	17,000.00	51,000.00	34,000.00	-
1 - C	24,000.00	12,000.00	(120,000.00)	36,000.00	12,000.00	36,000.00	-
Iteration 1 Post-allocation costs	41,000.00	22,000.00	56,000.00	133,000.00	203,000.00	295,000.00	750,000.00
2 - A	(41,000.00)	8,200.00	4,100.00	8,200.00	8,200.00	12,300.00	-
2 - B	2,200.00	(22,000.00)	6,600.00	2,200.00	6,600.00	4,400.00	-
2 - C	11,200.00	5,600.00	(56,000.00)	16,800.00	5,600.00	16,800.00	-
Iteration 2 Post-allocation costs	13,400.00	13,800.00	10,700.00	160,200.00	223,400.00	328,500.00	750,000.00
3 - A	(13,400.00)	2,680.00	1,340.00	2,680.00	2,680.00	4,020.00	-
3 - B	1,380.00	(13,800.00)	4,140.00	1,380.00	4,140.00	2,760.00	-
3 - C	2,140.00	1,070.00	(10,700.00)	3,210.00	1,070.00	3,210.00	-
Iteration 3 Post-allocation costs	3,520.00	3,750.00	5,480.00	167,470.00	231,290.00	338,490.00	750,000.00
4 - A	(3,520.00)	704.00	352.00	704.00	704.00	1,056.00	-
4 - B	375.00	(3,750.00)	1,125.00	375.00	1,125.00	750.00	-
4 - C	1,096.00	548.00	(5,480.00)	1,644.00	548.00	1,644.00	-
Iteration 4 Post-allocation costs	1,471.00	1,252.00	1,477.00	170,193.00	233,667.00	341,940.00	750,000.00
5 - A	(1,471.00)	294.20	147.10	294.20	294.20	441.30	-
5 - B	125.20	(1,252.00)	375.60	125.20	375.60	250.40	-
5 - C	295.40	147.70	(1,477.00)	443.10	147.70	443.10	-
Iteration 5 Post-allocation costs	420.60	441.90	522.70	171,055.50	234,484.50	343,074.80	750,000.00
6 - A	(420.60)	84.12	42.06	84.12	84.12	126.18	-
6 - B	44.19	(441.90)	132.57	44.19	132.57	88.38	-
6 - C	104.54	52.27	(522.70)	156.81	52.27	156.81	-
Iteration 6 Post-allocation costs	148.73	136.39	174.63	171,340.62	234,753.46	343,446.17	750,000.00
7 - A	(148.73)	29.75	14.87	29.75	29.75	44.62	0.01
7 - B	13.64	(136.39)	40.92	13.64	40.92	27.28	0.01
7 - C	34.93	17.46	(174.63)	52.39	17.46	52.39	0.00
Iteration 7 Post-allocation costs	48.57	47.21	55.79	171,436.40	234,841.59	343,570.46	750,000.02
8 - A	(48.57)	9.71	4.86	9.71	9.71	14.57	(0.01)
8 - B	4.72	(47.21)	14.16	4.72	14.16	9.44	(0.01)
8 - C	11.16	5.58	(55.79)	16.74	5.58	16.74	0.01
Iteration 8 Post-allocation costs	15.88	15.29	19.02	171,467.57	234,871.04	343,611.21	750,000.01



Appendix A: Excel Spreadsheet (Continued)

Iterations	Service Departments			Production Departments			
	A	B	C	1	2	3	
Iteration 8 Post-allocation costs	15.88	15.29	19.02	171,467.57	234,871.04	343,611.21	750,000.01
9 - A	(15.88)	3.18	1.59	3.18	3.18	4.76	0.01
9 - B	1.53	(15.29)	4.59	1.53	4.59	3.06	0.01
9 - C	3.80	1.90	(19.02)	5.71	1.90	5.71	-
Iteration 9 Post-allocation costs	5.33	5.08	6.18	171,477.99	234,880.71	343,624.74	750,000.03
10 - A	(5.33)	1.07	0.53	1.07	1.07	1.60	0.01
10 - B	0.51	(5.08)	1.52	0.51	1.52	1.02	(0.00)
10 - C	1.24	0.62	(6.18)	1.85	0.62	1.85	-
Iteration 10 Post-allocation costs	1.75	1.69	2.05	171,481.42	234,883.92	343,629.21	750,000.04
11 - A	(1.75)	0.35	0.18	0.35	0.35	0.53	0.01
11 - B	0.17	(1.69)	0.51	0.17	0.51	0.34	0.01
11 - C	0.41	0.21	(2.05)	0.62	0.21	0.62	0.02
Iteration 11 Post-allocation costs	0.58	0.56	0.69	171,482.56	234,884.99	343,630.70	750,000.08
12 - A	(0.58)	0.12	0.06	0.12	0.12	0.17	0.01
12 - B	0.06	(0.56)	0.17	0.06	0.17	0.11	0.01
12 - C	0.14	0.07	(0.69)	0.21	0.07	0.21	0.01
Iteration 12 Post-allocation costs	0.20	0.19	0.23	171,482.95	234,885.35	343,631.19	750,000.11
13 - A	(0.20)	0.04	0.02	0.04	0.04	0.06	-
13 - B	0.02	(0.19)	0.06	0.02	0.06	0.04	0.01
13 - C	0.05	0.02	(0.23)	0.07	0.02	0.07	-
Iteration 13 Post-allocation costs	0.07	0.06	0.08	171,483.08	234,885.47	343,631.36	750,000.12
14 - A	(0.07)	0.01	0.01	0.01	0.01	0.02	(0.01)
14 - B	0.01	(0.06)	0.02	0.01	0.02	0.01	0.01
14 - C	0.02	0.01	(0.08)	0.02	0.01	0.02	-
Iteration 14 Post-allocation costs	0.03	0.02	0.03	171,483.12	234,885.51	343,631.41	750,000.12
15 - A	(0.03)	0.01	-	0.01	0.01	0.01	0.01
15 - B	-	(0.02)	0.01	-	0.01	-	0.00
15 - C	0.01	-	(0.03)	0.01	-	0.01	-
Iteration 15 Post-allocation costs	0.01	0.01	0.01	171,483.14	234,885.53	343,631.43	750,000.13
16 - A	(0.01)	-	-	-	-	-	(0.01)
16 - B	-	(0.01)	-	-	-	-	(0.01)
16 - C	-	-	(0.01)	-	-	-	(0.01)
Iteration 16 Post-allocation costs	-	-	-	171,483.14	234,885.53	343,631.43	750,000.10