How Unexpected Changes in the CPI and the PPI Affect Expected Inflation and Forward Rates

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Abstract

This paper examines how unexpected changes in the consumer price index (CPI) affect both spot rates and forward rates. It finds that unexpected changes in the CPI affect inflation expectations as far as seven years into the future. Additionally, the paper finds that the unexpected changes in the producer price index (PPI) do not affect either the spot rates or forward rates.

1. Introduction

This paper examines the extent to which unexpected changes in the consumer price index (CPI) and the producer price index (PPI) affect inflation expectations. It is widely reported in the financial press that large unexpected changes in the CPI and PPI signal that future inflation will also be different. For example, a large unexpected increase in the CPI is interpreted as indicating higher inflation in the future. When announced unexpected changes in the CPI and PPI are coincident with changes in bond prices, stock prices, or commodity prices, the price changes are often attributed to a change in inflationary expectations arising from the CPI or PPI announcement.

The Fisher hypothesis (identified by Irving Fisher (1896)) suggests a method for testing the extent to which unexpected changes in the CPI and the PPI have implications for future inflation. If the market believes that an unexpected increase in the CPI portends higher inflation in the future as well, then bonds will decline and interest rates will increase. Thus, if unexpected increases in the CPI and the PPI were associated with increases in interest rates, this would indicate that CPI and PPI announcements convey information regarding future inflation to the market. By disaggregating the term structure into forward rates, and relating changes in the forward rates to surprise changes in the CPI and PPI, this paper infers how far into the future interest rates (and inflation expectations) are affected by the changes.

The CPI and PPI figures are released approximately two weeks into each month. The indices report the Bureau of Labor Statistic's price survey conducted the previous month. Thus, both the CPI and PPI are lagging indicators that report price levels that were measured up to 45 days earlier. How is it that a clearly lagging indicator can be so widely viewed as a leading indicator with lead time times as far as several years? One explanation is that inflation is viewed not as a pure random walk, but as a chronic malady that over time has a tendency to spiral either up or down. Given time, price increases in one industry will cascade over into other industries affecting future inflation. Also, there is a feedback mechanism that allows price increases in industry X to spill into other industries and thereby affect the prices in industry X at a later time. Accordingly, an unexpected increase or decrease in the inflation measure can have implications for at least the current month and the near future. This paper examines how far into the future are interest rates (forward rates) affected by unexpected changes in the announced CPI and PPI indices.

The PPI has some advantages and disadvantages over the CPI as a predictor of future inflation. The PPI measures costs at the wholesale level. If costs increase at the wholesale level and continue increasing, then the building inflationary pressure at the wholesale level will eventually cause price increases at the retail level as well. Thus, PPI not only measures past wholesale price increases but also may indicate likely future CPI increases. Another advantage that the PPI has over the CPI is that it is released from one to four days prior to the CPI. If both indicate higher future inflation, then the PPI will signal the market first, perhaps making the CPI announcement less informative. The PPI also has some disadvantages. First it excludes the price of services. Second, it is seen as more volatile than the CPI and therefore less reliable. One issue examined in this paper is the comparison of the PPI and the CPI as leading indicators of future inflation.

Earlier studies have examined how surprise changes in the CPI and PPI affect bond prices or interest rates with mixed results. Papers finding significant coefficients associated with both the CPI and PPI are Hess (2004), Green (2004), Balduzzi, Elton, and Green (2001), Fleming and Remolona (1999) and Edison (1996). Varedas (2006) finds a significant reaction in the bond futures markets to announcements regarding both CPI and PPI. On the other hand, Urich and Wachtel (1984) find CPI coefficients not significant and PPI coefficients weakly significant. Smirlock (1986) finds the CPI and PPI coefficients not significant prior to October 1979 but significant thereafter. Dwyer and Hafer (1989) find that neither T-Bills nor

20 year bonds react to unexpected changes in the CPI; however, PPI has a significant impact on 30 year bond rates but not T-Bills.

This paper extends the previous research in several ways. The earlier studies examine much shorter periods than this study. Five of the studies examine a period of 5 years or less. The longest period is 9 years and 1 month examined by Veredas (2006). This study examines the 21 year period from March 1988 through February 2009. Additionally, none of the previous papers attempt to identify how far into the future forward interest rates are affected. This paper addresses this issue by disaggregating the forward rates out of the yield curve and relating the forward rates to the surprise changes in the CPI and PPI.

This paper finds that unexpected changes in the CPI have a significant impact on the yields of bonds with maturities as far into the future as 10 years. The unexpected changes in the CPI affect forward rates as far as seven years in to the future. The paper concludes that the market reacts to unexpected changes in the CPI by revising inflation expectations as far as seven years into the future. With regard to the PPI this paper shows that there is no significant relationship between unexpected changes in the PPI and bond yields or forward rates.

2. Test Methodology and Results

A. Unexpected CPI and PPI and spot rates.

This paper examines the impact of unexpected changes in the consumer price index and the producer price index on interest rates. The consumer price index (CPI-U, seasonally adjusted) and the producer price index (PPI) are two of the most widely reported and used inflation measures. The CPI is a measure of the change in retail prices while the PPI is a measure of the changes in prices at the wholesale level. The PPI is generally released from one to four days before the CPI is reported. The expected levels of the indicies are taken from surveys of economists that were compiled on the Friday of the week before the announcement and are published in the Wall

Street Journal (March 1988 through March 2006) or Barron's (after March 2006). In this paper, the unexpected changes in the indices are estimated by subtracting the expected level of the CPI and the PPI from the actual reported level. This study uses monthly CPI and PPI announcements over the 21-year period from March 1988 through February 2009.[1]

[1] The Wall Street Journal began publishing the MMS consensus forecasts in February 1988. After March 1992 the Wall Street Journal switched to Technical Data Corporation (see Schirm (2003). In March 2006 the Wall Street Journal stopped publishing the consensus forecast. This study uses estimates published in Barron's after that date. This study covers the period from March 1988 through Feb 2009, a period of 252 months. However, the number of months in the study is only 251. A US government shutdown because of budget problems combined with a paralyzing winter storm in the Washington D.C. area during the second week in January 1996 caused a delay in the reporting of the CPI for December 1995. Accordingly, the *Wall Street Journal* did not estimate the expected CPI and PPI for that month.

Table 1: Frequency of unexpected changes in the CPI and the PPI.

Panel A: Frequency table for the unexpected changes in the CPI

UCPI	Frequency	Percent	Cumulative Frequency	Cumulative Percent
-0.5	1	0.40	1	0.40
-0.4	3	1.20	4	1.59
-0.3	5	1.99	9	3.59
-0.2	19	7.57	28	11.16
-0.1	72	28.69	100	39.84
0	77	30.68	177	70.52
0.1	53	21.12	230	91.63
0.2	13	5.18	243	96.81
0.3	5	1.99	248	98.80
0.4	2	0.80	250	99.60
0.5	1	0.40	251	100.00

Panel B: Frequency of unexpected changes in the PPI.

UPPI	Change in Frequency	Percent	Cumulative Frequency	Cumulative Percent
-1.3	1	0.40	1	0.40
-1.1	3	1.20	4	1.59
-1	2	0.80	6	2.39
-0.8	1	0.40	7	2.79
-0.7	1	0.40	8	3.19
-0.6	3	1.20	11	4.38
-0.5	14	5.58	25	9.96
-0.4	11	4.38	36	14.34
-0.3	30	11.95	66	26.29
-0.2	20	7.97	86	34.26
-0.1	28	11.16	114	45.42
0	35	13.94	149	59.36
0.1	34	13.55	183	72.91
0.2	18	7.17	201	80.08
0.3	11	4.38	212	84.46
0.4	11	4.38	223	88.84
0.5	7	2.79	230	91.63
0.6	6	2.39	236	94.02
0.7	4	1.59	240	95.62
0.8	4	1.59	244	97.21
0.9	2	0.80	246	98.01
1.1	2	0.80	248	98.80
1.2	1	0.40	249	99.20
1.4	1	0.40	250	99.60
1.6	1	0.40	251	100.00

Note: UCPI = ACPI – ECPI and UPPI = APPI – EPPI where UCPI and UPPI are the unexpected changes in the Consumer Price Index (CPI-U, seasonally adjusted) and the PPI; ACPI and APPI are the actually reported change in the CPI and PPI; and ECPI and EPPI are the consensus expected change in the CPI and PPI.

Table 1, Panel B shows the frequency of unexpected changes in the two indices. The unexpected changes in the CPI range from an unexpected decline of .5% through an unexpected increase of .5%. Of the 251 CPI announcements, 175 (81.4%) announce unexpected changes in the CPI of .1% or less. On the other hand, the unexpected changes in the PPI range from an unexpected decline of 1.3% through an unexpected increase of 1.2%. Only 90 of the 214 PPI announcements (42%) were for unexpected changes in the PPI of .1% or less. The expectations for the CPI changes were much closer to the actual reported changes than were the expected PPI changes to the actual reported PPI changes.

The term structure of interest rates was drawn from the federal reserve H.15 statistical release. The H.15 release contains the Treasury Yields for 1 year, 2 years, 3 years, 5years, 10 years, and 20 years. The yields are based on composite quotes reported by U.S. Government securities dealers to the Federal Reserve Bank of New York.[2] To obtain the constant maturity yields, personnel at treasury construct a yield curve each business day and yield values are then read from the curve at fixed maturities.[3] The 20 year maturity series began in Oct 1993. The impact of the unexpected change in the CPI and the PPI was measured using the regression models [1] and [2] for each maturity strata.

$$\Delta R_t = \alpha + \beta\, U\Delta CPI_t + \gamma\, ECPI_t + e_t \qquad [1]$$

$$\Delta R_t = \alpha + \beta\, U\Delta PPI_t + \gamma\, EPPI_t + e_t \qquad [2]$$

In the above regressions, ΔR_t is the change in the yield quote from the previous day, $U\Delta CPI_t$ is the unexpected change in the consumer price index , $U\Delta PPI_t$ is the unexpected change in the producer price index and $ECPI_t$ and $EPPI_t$ are the expected changes in the CPI and the PPI. Notice that the equations in [1] and [2] separate the actual change in the indices into two parts, the expected change and the unexpected change.

[2] The term structure is the set of annual yields on riskfree zero coupon bonds (sometimes called spot rates). The H.15 release is calculated using coupon bonds. Accordingly, the yield curve in the H.15 release is only an approximation of the true term structure.

[3] The H.15 release can be found at the federal reserve internet site: Http://www.federalreserve.gov/releases/h15/data.htm

Table 2: The relationship between spot rates and unexpected changes in the CPI and the PPI.

Panel A: Unexpected Change in the Consumer Price Index[1]

Spot Rates	n	α	α t-stat	β	β t-stat	γ	γ t-stat	Adj R-sq[2]	F stat
1 year	251	-.005	-0.85	.088	2.94**	-.010	-0.54	.037	4.35*
2 year	251	-.002	-0.26	.095	2.79**	-.012	-0.60	.023	3.91*
3 year	251	-.001	-0.15	.103	2.97**	-.013	-0.60	.027	4.43*
5 year	251	-.002	-0.23	.104	3.05**	-.009	-0.43	.029	4.72**
7 year	251	-.002	-0.30	.105	3.22**	-.010	-0.48	.033	5.22**
10 year	251	-.002	-0.38	.095	3.07**	-.008	-0.45	.029	4.77**
20 year	184	-.006	-0.90	.069	1.93	.006	0.31	.015	2.36

Panel B: Unexpected Change in the Producer Price Index[1]

Spot Rates	n	α	α t-stat	β	β t-stat	γ	γ t-stat	Adj R-sq[2]	F stat
1 year	251	-.003	-0.75	.018	1.77	-.017	-1.75	.010	2.22
2 year	251	-.002	-0.43	.012	0.95	-.014	-1.08	.000	0.74
3 year	251	-.005	-0.95	.010	0.77	-.012	-0.96	.000	0.55
5 year	251	-.003	-0.61	.010	0.85	-.017	-1.44	.001	1.08
7 year	251	-.004	-0.80	.010	0.84	-.016	-1.37	.000	0.99
10 year	251	-.006	-1.18	.013	1.18	-.010	-0.92	.000	0.81
20 year	184	-.004	-0.86	.009	0.84	.003	0.25	.000	0.60

Notes:

1. The following are used in the above table:

$$\text{Panel A:} \quad \Delta R_t = \alpha + \beta\, U\Delta CPI_t + \gamma\, ECPI_t + e_t$$
$$\text{Panel B:} \quad \Delta R_t = \alpha + \beta\, U\Delta PPI_t + \gamma\, EPPI_t + e_t$$

ΔR_t is the change in the yield quote from the previous day, $U\Delta CPI_t$ is the unexpected change in the consumer price index , $U\Delta PPI_t$ is the unexpected change in the producer price index and $ECPI_t$ and $EPPI_t$ are the expected changes in the consumer price index and the producer price index. The equations separate the actual change in the indices into two parts, the expected change and the unexpected change. The regressions examine the period March 1988 through February 2009.

2. If adjusted R-squared is negative then it is reported as zero.

* Significant at the .05 level (two sided).
** Significant at the .01 level (two sided).

Table 2 Panel A shows that the t-statistics for the β coefficient for the UΔCPI announcements are significantly positive at all maturity levels up to ten years. This indicates that the spot rates react to unexpected changes in the CPI is as predicted by the Fisher equation. Unexpected increases in the CPI indicate higher current and future inflation and are associated with increases in the spot rates, and unexpected decreases in the CPI are associated with decreases in expected inflation and lower spot rates. The results are consistent with the observation that unexpected increase in the

CPI are interpreted by the market to indicate that inflation will be higher than expected in the future and unexpected decreases are interpreted by the market to indicate that inflation will be lower in the future.

Table 3 Panel B, shows that the β coefficient for the UΔPPI is not significantly different from zero. Additionally the adjusted r-squared for the regressions are less than .01 in every test. The failure to find significant coefficients and the low r-squared statistic indicates that unexpected changes in the producer price index have no effect on the term structure. Accordingly, the results indicate that the announced PPI level does not contain new information regarding future inflation. It is helpful to observe that not only are the regression coefficients not significant but in four of the six regressions the coefficients have the wrong sign. Accordingly, the findings with regard to the PPI is not simply a result of the failure to find significance because of a higher standard deviation arising from the greater volatility of the PPI. If the sign of the coefficients were all positive then the high standard deviation could result in a Type 1 error. The negative coefficients rule this out as an explanation for failure to find significance.

B. Forward rates

In the previous section it was shown that unexpected changes in the CPI are associated with significant shifts in interest rates for all maturity levels up to ten years. The 20 year maturity spot rates had a positive sign but was not significant. As we mentioned above, this finding indicates that the CPI announcement conveys new information regarding future inflation to the market.

Although the spot rates are significantly greater across the range of maturities up to ten years, we cannot infer from these results that the CPI contains information about inflation ten years from now. Even if the expected inflation for just one year is higher, the spot rates will increase across all future maturities. To see this, consider for example the two year spot rate. Assume that the market is a pure expectations market such that an investment for two periods is expected to give the same return as an investment for one period that is rolled over for an additional period.

Equation [3] shows the relationship between the one year spot rate and the two year spot rate in such a market.

$$(1+{}_0r_1)(1+{}_1r_2) = (1+{}_0r_2)^2 \qquad [3]$$

Where ${}_0r_1$ is the one year spot rate, ${}_0r_2$ is the two year spot rate, and ${}_1r_2$ is the expected one year spot for the period starting at t=1 through t=2. Now suppose that there is a large unexpected change in CPI that causes the market

to anticipate higher inflation in year 1 but that it has no impact on inflation expectations in year 2 and beyond. The Fisher equation will predict that $_0r_1$ will increase. However, since $_0r_2$ is a function of $_0r_1$, then $_0r_2$ will increase even if $_1r_2$ remains unchanged. Even if inflation does not affect future interest rates after period 1, the spot rates for periods longer than 1 yer will be affected. This result can be generalized to show that a change in inflationary expectations in year 1 will affect all future spot rates. Accordingly, simply showing that spot rates as far as 10 years out are affected by unexpected changes in interest rates does not imply that inflationary expectations regarding future inflation are affected ten year out as well.

A research question addressed by this paper is how far into the future are inflation expectations affected by unexpected changes in the CPI and the PPI. To examine this, we test the extent to which forward rates are affected by the unexpected changes. Equation [4] is a general form of equation [3].

$$(1+{}_0r_{n-1})^{n-1}(1+{}_{n-1}r_n) = (1+{}_0r_n)^n \qquad [4]$$

The spot rates $_0r_{n-1}$ and $_0r_n$ can be obtained from the term structure and are known at time t=0. The term $_{n-1}r_n$ is the one period spot rate from time n-1 to time n. This is not known at time zero. However, equation [4] can be solved to obtain $_{n-1}r_n$. When the implied future spot rate is obtained this way it is typically called the forward rate and written as $_{n-1}f_n$. Equation [5] shows a general form for calculating the forward rate.

$$_{n-1}f_n = \left(\frac{(1+{}_0r_n)^n}{(1+{}_0r_{n-1})^{n-1}}\right) - 1 \qquad [5]$$

In a pure expectations market the forward rate will be equal to the expected future spot rate. By disaggregating the forward rates out of the spot rates we can see if the forward rates are affected by the unexpected

changes in the CPI. Thus, we can determine how far into the future unexpected changes in the CPI affect inflation expectations.[4]

Appendix A explains how the implied forward rates were extracted from the available spot rates. The forward rates that cover periods more than a year are annualized (adjusted to one year) by extracting the appropriate geometric average. The forward rates were regressed against the unexpected changes in the CPI and the PPI with the results given in table 4. The tests were performed using equation [6] for the CPI and equation [7] for the PPI.

$$\Delta f_t = \alpha + \beta\, U\Delta CPI_t + \gamma\, ECPI_t + e_t \qquad [6]$$

$$\Delta f_t = \alpha + \beta\, U\Delta PPI_t + \gamma\, EPPI_t + e_t \qquad [7]$$

The independent variables are as described above for equations [1] and [2]. The dependent variable Δf_t is the change in the forward rate from the day before the announced change in the CPI or PPI to the day of the announced change.

[4] This test does not require the assumption of the term structure pure expectation hypothesis to infer the impact of unexpected changes in the CPI or the PPI. Suppose the term structure is influenced by a liquidity preference or a preferred habitat effect. A change in the expected future spot rate will still affect the expected future spot rate and should therefore have an impact on the forward rate.

Table 3: The relationship between forward rates and unexpected changes in the CPI and the PPI.

Panel A: Unexpected Change in the Consumer Price Index[1]

Forward Rates Begin Year	End Year	n	α	α t-stat	β	β t-stat	γ	γ t-stat	Adj R-sq[2]	F stat
1	2	251	.002	0.18	.101	2.32*	-.015	-.57	.013	2.70
2	3	251	.000	0.05	.118	2.87**	-.013	-0.52	.025	4.14*
3	5	251	-.003	-0.32	.107	2.81**	-.003	-0.14	.024	4.12*
5	7	251	-.003	-0.38	.107	2.87**	-.011	-0.49	.024	4.13*
7	10	251	-.003	-0.51	.069	2.23*	-.006	-0.29	.012	2.51
10	20	184	-.005	-0.79	.049	1.37	.017	0.85	.010	1.88

Panel B: Unexpected Change in the Producer Price Index[1]

Forward Rates Begin Year	End Year	n	α	α t-stat	β	β t-stat	γ	γ t-stat	Adj R-sq[2]	F stat
1	2	251	-.001	-0.20	.006	0.37	-.010	-0.58	.000	0.18
2	3	251	-.010	-1.75	.005	0.33	-.008	-0.57	.000	0.17
3	5	251	.000	-0.06	.012	0.86	-.026	-1.96	.007	1.92
5	7	251	-.006	-1.08	.009	0.69	-.013	-0.95	.000	0.51
7	10	251	-.010	-1.88	.021	1.75	.003	0.31	.009	2.13
10	20	251	-.002	-0.36	.014	1.38	-.007	-0.65	.000	0.95

Notes:

1. This table presents the OLS estimation of the following:

Panel A: $\Delta F_t = \alpha + \beta\ U\Delta CPI_t + \gamma\ ECPI_t + e_t$

Panel B: $\Delta F_t = \alpha + \beta\ U\Delta PPI_t + \gamma\ EPPI_t + e_t$

Where ΔF_t is the change in the forward rate from the previous day and ΔU_t is the unexpected change in the CPI (panel A) and the PPI (panel B). , $U\Delta CPI_t$ is the unexpected change in the consumer price index , $U\Delta PPI_t$ is the unexpected change in the producer price index and $ECPI_t$ and $EPPI_t$ are the expected changes in the consumer price index and the producer price index.

2. If adjusted R-squared is less than zero it is reported as zero.

* Significant at the .05 level (two sided).
** Significant at the .01 level (two sided).

Panel A of Table 3 shows the results of estimating the models in equation [6]. The significance of the β term indicates that the forward rates up to year 7 have significant coefficients. The coefficient for the regression for forward rates from year 7 through year 10 is significant but the F-statistic for the regression is not significant. The coefficient for year10 through year 20 has the correct sign but is not significant at the 5% level. The regression for the forward rate from year 1 through year 2 has a significant β coefficient, but the F-statistic is not significant.

Panel B of Table 5 shows that when the unexpected change in PPI is used to explain forward rates, the coefficients and the F-statistics are insignificant.[5]

The results indicate that unexpected changes in the CPI affect inflation expectations far into the future. The results indicate that expectations change as far as seven years into the future.

3. Summary and Conclusion

The financial press often reports that announced changes in the consumer price index and the producer price index have implications for future inflation. Accordingly, These two indicators are used by both financial analysts and academics as a leading indicator of future inflation. This paper tests the extent to which the CPI and PPI affect market perception of future inflation. Surprise changes in the PPI have no significant impact on either the spot rates or forward rates. These results are contrary to earlier studies that find that unexpected changes in the PPI affect bond prices and/or spot rates. The earlier studies examine much shorter time periods than this study. This study examine a 21 year period.

The paper shows that unexpected changes in the CPI cause significant changes in spot rates at least ten years into the future. Additionally, unexpected changes in the CPI affect forward rates as far as seven years into the future. This has the surprising implication that recent unexpected changes in the CPI affect inflation expectations as far as seven years into the future.

[5] The Federal Reserve targets (and to some extent controls) the federal funds rate and the federal funds rate is closely correlated with the T-Bill and the short term treasury bond market. Accordingly, the very short term treasury bond market is less likely to be affected by inflation expectation than by Federal Reserve policy. This could account for the low F-statistic for the forward rate beginning in year 1 and ending in year 2.

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Appendix A

The forward rates used in the paper were calculated from the spot rates as shown below. In the discussion, ${}_0r_i$ indicates the spot rate at time 0 for debt maturing at the end of i years.

1. The forward rate from the end of year 1 through the end of year 2 is:

$$ {}_1 f_2 = \left(\frac{(1 + {}_0 r_2)^2}{(1 + {}_0 r_1)} \right) - 1 $$

2. The forward rate from the end of year 2 through the end of year 3 is:

$$ {}_2 f_3 = \left(\frac{(1 + {}_0 r_3)^3}{(1 + {}_0 r_2)^2} \right) - 1 $$

3. The geometric average annual forward rate for years 4 and 5 is:

$$ {}_3 f_5 = \left(\frac{(1 + {}_0 r_5)^5}{(1 + {}_0 r_3)^3} \right)^{1/2} - 1 $$

6. The geometric average annual forward rate for years 6 and 7 is:

$$ {}_5 f_7 = \left(\frac{(1 + {}_0 r_7)^7}{(1 + {}_0 r_5)^5} \right)^{1/2} - 1 $$

7. The geometric average annual forward rate for the years 8, 9, and 10 is:

$$ {}_7 f_{10} = \left(\frac{(1 + {}_0 r_7)^{10}}{(1 + {}_0 r_5)^7} \right)^{1/3} - 1 $$

8. The geometric average annual forward rate for the ten year period beginning after year 10 is:

$$ {}_{10} f_{20} = \left(\frac{(1 + {}_0 r_{20})^{20}}{(1 + {}_0 r_{10})^{10}} \right)^{1/10} - 1 $$