A Comparison of Control Charting Applications for Variable Sample Sizes in Service Processes

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Abstract

Over the last several decades, quality improvement approaches and applications have expanded beyond the manufacturing floor to other areas of the operation. In more recent years, service industries have embraced the concepts of continuous improvement and six sigma as well. In applying the quality tools to service processes, challenges have developed in making the transition from factory volume production rates and the sometimes more variable activity rates of services.

A problem seen when applying control charts to monitor performance for service processes is that the size of the sample can vary dramatically from sample to sample. For p-chart and u-chart applications, several transformation methods for handling the variation in the size of the sample have been developed over the past few decades.

When faced with the desire to take advantage of run rules and the straight line control limits under process conditions of varying sample sizes, the practitioner has several choices. In practice, having one transformation that is a better performer than others under given process conditions will allow the practitioner the ability to use the more appropriate transformation method, and thus have a more optimal control charting application.

Introduction

Service industry processes are being identified more readily as potential applications of control charts and other statistical analysis tools of continuous improvement (Woodall, 2006; Chow, Woodford, & Showers-Chow, 2008; Chow, Finney, & Woodford, 2009; Duran & Albin, 2010; Kumar, 2005; and Gregorio & Cronemyr, 2011). Quality improvement methods are being adapted and converted from manufacturing applications to those processes in services including customer calls to service call centers (Arora, & Bandara, 2006; Avramidis, Chan, & L'Ecuyer, 2009; and Landon, Ruggeri, Soyer, & Tarimcilar, 2010), service quality in banking (Chen, 2009), patient care in medical clinics (Goodnough, et al, 2011; Joner, Woodall, Reynolds, & Fricker, 2008; Boe, Riley, & Parsons, 2009), and student success rates in higher education courses (Murgatroyd, 1993).

In this study, three transformation methods for p-charts and u-charts when sample sizes vary are compared using simulation and Pitman Closeness Criterion. The results provide process conditions under which each transformation method performs better than the others. However, the results also reveal that in real world applications, these differences may be negligible. The results of the study serve as a reminder that when using simulations and computer computations, a review of the data is necessary to determine real relevance in the findings.

Attribute control charts have been used in industry for decades to track and identify special cause variation. One challenge to the quality practitioner is situations in which sample sizes vary from sample to sample. The desire (or contractual requirement) to plot daily process data can create this special case of the attribute control chart. Quality control texts provide a few methods to overcome part of the problem with variable sample size (Grant & Leavenworth, 1988; Montgomery, 1991), though in application these often do not provide a ready solution without adding additional concerns. For example, some quality control texts suggest the use of variable control limits or the straight line control limit (Grant & Leavenworth, 1988; Montgomery, 1991). However, use of the variable control limits or the straight line control limits derived using the average sample size can create more problems than they seem to solve.

The variable control limit method adjusts for the change in sample size by recalculating the upper and lower control limits for each sample. Since the control limits are based on the actual size of the sample, they cannot be calculated prior to collection of the sample, so these calculations become part of the daily activity (Grant & Leavenworth, 1988). This method also gives a "skyline" effect on the chart and as Montgomery (1991) points out, identifying trends can be difficult and looking at runs and nonrandom patterns is of little use. In cases where sample size varies substantially, this can cause problems for personnel responsible for the production process, as it creates a continually changing set of control limits.

The straight-line control limit method utilizes control limits based on the average sample size. Technical considerations of the straight-line control limits suggest that a 20-25% variation in sample size is acceptable (Grant and Leavenworth, 1988). Using this method, when points are near the control limits, either just inside or just outside, further review of the samples is required to determine if special cause variation is present, since the control limits are only approximated based on the average sample size. This requires additional effort of employees responsible for the operation of the process, and increased management oversight to assure that the process is correctly being monitored.

When faced with the desire to take advantage of run rules and the straight line control limits under process conditions of varying sample sizes, the practitioner has several choices. Duncan (1948) proposed a normalized transformation for p-charts with varying sample size. Soffer (1981) pointed out the practical limitations of the stabilized p-chart which plots in standardized Z scores instead of proportion nonconforming, which are more readily understood by production personnel. He proposed a transformed p-chart which plots in the identifiable proportion nonconforming. Both cases lead to a simple adaptation for u-charts with varying sample size, the former provided in Montgomery (1991).

Chan and Xiao (1990) and Rocke (1990) submitted similar transformations for both the p-chart and the u-chart. These u-chart transformations are based on the normal distribution, plot in nonconformances per unit, and maintain straight line control limits for ease of use on the production floor.

Plotting stabilized p gives the ease of the straight-line control limits while maintaining the ability to effectively identify any adverse trends. The main detractor for using stabilized p is that the plotted points are in terms of Z-values and not in the proportion nonconforming. This can often cause confusion for shop floor personnel responsible for the production process.

The Alternate Transformations

As a way of taking advantage of the straight-line control limits found with the stabilized p, and alleviating the problem of plotting Z-values, Soffer (1981) proposed a transformation, which converts back to a more recognizable proportional value. Rocke (1990) and Chan and Xiao (1990) proposed additional adjusted and transformed p-charts (as well as u chart transformations), which also plot in proportion nonconforming units of measure.

This study uses simulation and Pitman Criterion as a method of evaluating the effectiveness of these transformations. Duncan's stabilized p is used as the "true" value of the transformed p. Of particular interest is to identify if, under differing conditions of proportion nonconforming and/or sample size, one transformation outperforms the others (in a Pitman sense). Knowing this would allow the practitioner the ability to implement a transformation method most suitable to their needs.

Transformation Formulas

Tables 1 and 2 provide the transformation formulas used to calculate the plotted point, standard error, center line and control limits for each of the p-chart and u-chart transformations evaluated.

p-Chart Transformations

Method	Transformation Plotted	Standard Error	Center Line & Control Limits
Duncan	$p_{stablized} = \frac{p - \bar{p}}{\sqrt{n}}$	$\sigma_{stabilized} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$CL = 0$ $Limit = \pm 3$
Soffer	$y = x / \sqrt{n} - \bar{p}\sqrt{n}$	$\sigma_y = \sqrt{\bar{p}(1-\bar{p})}$	$CL = 0$ $Limit = \pm 3\sqrt{\bar{p}(1-\bar{p})}$
Rocke	$p^{adj} = \bar{p} + \frac{x_i - n_i \bar{p}}{\sqrt{n_i \bar{n}}}$	$\sigma_{padj} = \sqrt{\bar{p}(1-\bar{p})/\bar{n}}$	$CL = \bar{p}$ $Limit = \bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$
Chan/ Xiao	$p_i{*} = \bar{p} + \sqrt{\frac{n_i}{\bar{n}}}\left(p_i - \bar{p}\right)$	$\sigma_{p*} = \sqrt{\bar{p}(1-\bar{p})/\bar{n}}$	$CL = \bar{p}$ $Limit = \bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/\bar{n}}$

Table 1

u-Chart Transformations

Method	Transformation Plotted	Standard Error	Center Line & Control Limits
Duncan	$u_{stablized} = \frac{u - \bar{u}}{\sqrt{\bar{u}/n}}$	$\sigma_{stabilized} = \sqrt{\bar{u}/n}$	$CL = 0$ $Limit = \pm 3$
Soffer	$y_u = \frac{c}{\sqrt{n}} - \bar{u}\sqrt{n}$	$\sigma_y = \sqrt{\bar{u}}$	$CL = 0$ $Limit = \pm 3\sqrt{\bar{u}}$
Rocke	$u^{adj} = \bar{u} + \frac{c_i - n_i\bar{u}}{\sqrt{n_i\bar{n}}}$	$\sigma_{uadj} = \sqrt{\bar{u}/\bar{n}}$	$CL = \bar{u}$ $Limit = \bar{u} \pm 3\sqrt{\bar{u}/\bar{n}}$
Chan/ Xiao	$u_i{*} = \bar{u} + \sqrt{\frac{n_i}{\bar{n}}}\left(u_i - \bar{u}\right)$	$\sigma_{u*} = \sqrt{\bar{u}/\bar{n}}$	$CL = \bar{u}$ $Limit = \bar{u} \pm 3\sqrt{\bar{u}/\bar{n}}$

Table 2

The Comparison

In this study, we concern ourselves with the deviation from the upper control limit (UCL) and the lower control limit (LCL). Studying these deviations at the control limits is based on two concepts. First, Shewhart (1931) only provided “rules” regarding points plotted beyond the upper or lower control limits. Additional run rules, which have been developed and reported by many, have relevance in application and should be incorporated in the actual use of the transformed control chart but are not considered for the purpose of this study. The second concept is that performance at the control limits seems of logical importance, as points above the UCL would indicate potential process degradation, and points below the LCL would signal an improving process. Both cases would require further investigation and are of practical importance in process monitoring.

While Ryan & Woodall (2010) and others have employed average run length to compare performance of different control charts, Chow et al (2007) proposed using Pitman criteria for comparing control chart performance. Pitman (1937) introduced his method of determining the closer estimator based on the probability of the absolute deviation from the true parameter. Using Pitman Criterion, if $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimators of θ, then $\hat{\theta}_1$ is said to be Pitman Closer than $\hat{\theta}_2$ if

$$(\Pr(|\hat{\theta}_1 - \theta| < |\hat{\theta}_2 - \theta|) > \tfrac{1}{2}.$$

Keating, Mason, and Sen (1993), among others, have presented additional concepts including the concept of nearness. $\hat{\theta}_1$ is Pitman Nearer (PN) than $\hat{\theta}_2$ if

$$\Pr\left(\left|\hat{\theta}_1 - \theta\right| < \left|\hat{\theta}_2 - \theta\right|\right) > \Pr\left(\left|\hat{\theta}_1 - \theta\right| > \left|\hat{\theta}_2 - \theta\right|\right)$$

Pitman realized the potential for intransitivity with his measure of closeness, which is a possible concern of this study. When comparing three estimators, a possibility exists that $\hat{\theta}_1 \,\mathrm{PN}\, \hat{\theta}_2$, $\hat{\theta}_2 \,\mathrm{PN}\, \hat{\theta}_3$, and $\hat{\theta}_3 \,\mathrm{PN}\, \hat{\theta}_1$.

A point that needs to be addressed is that Pitman Criterion only looks at which estimator is closer (or nearer) to the true value. Detractors of this method of comparison would point out the potential lack of relevant difference in the Pitman comparison (Fountain, Keating, & Maynard, 1996). This concern can become a real problem when using computer simulation and calculations in the comparison. In particular, when looking at the absolute difference between two values, the computer will look at the actual calculated values in making the comparison, without regard to significant digits or relevant decimal places.

To address this concern, a follow-up evaluation makes the Pitman comparisons after rounding the deviations to the 4th decimal place, as this would represent the millionth decimal place, which should be suitable for a study of this nature. In defining the closest or best estimator, Pitman (1937) pointed out that the practicality of the best or closest estimator needs to be considered, and the actual application of the estimator as well as the penalty for mistake should be considered.

p-Chart Comparison

The simulation set the sample size as a uniform random variable and the proportion nonconforming as binomially distributed. In order to evaluate the performance of each transformation under various conditions, the sample sizes and proportions are varied. Sample sizes used in the simulation were in uniform ranges of 100 - 1000, 1000 - 5000, and 5000 - 15000. Proportion nonconforming were set at 0.001, 0.0001, and 0.00001. An initial simulation run of 200 trials established the estimates for $\bar{p}$ and $\bar{n}$. The number of "defects" per unit was produced using c as a Poisson random variable and u as the average non-conformances per unit inspected. Under this study, c was randomly generated at averages of 1.0, 1.5, 2.0, 2.5, and 3.0. An initial simulation run of 200 trials was used to establish process averages on which the control limits would be based.

Control limits for each estimator of p were calculated based on the estimates for $\bar{p}$ and $\bar{n}$ in the initial simulation run. The main simulation consisted of 10,000 trials for each combination range of p and n. With each trial, the deviation from both the UCL and LCL were calculated. Table 3 provides the calculations for deviation from the upper and lower limits for each transformation. Table 4 shows the calculations for determining the absolute deviation from the "true" estimator value of the stabilized p transformation. In order to make this comparison, deviations are converted using the standard error of each transformation so that all comparisons are in terms of standard deviation.

Deviations from Limits

Method of Transformation	Deviation from Upper Limit	Deviation from Lower Limit
Duncan Z	$\theta_{UCL} = UCL_Z - Z$	$\theta_{LCL} = Z - LCL_Z$
Soffer	$\hat{\theta}_{1UCL} = UCL_{Soffer} - p_{Soffer}$	$\hat{\theta}_{1LCL} = p_{Soffer} - LCL_{Soffer}$
Rocke	$\hat{\theta}_{2UCL} = UCL_{Rocke} - p_{Rocke}$	$\hat{\theta}_{2LCL} = p_{Rocke} - LCL_{Rocke}$
Chan/Xiao	$\hat{\theta}_{3UCL} = UCL_{Chan/Xiao} - p_{Chan/Xiao}$	$\hat{\theta}_{3LCL} = p_{Chan/Xiao} - LCL_{Chan/Xiao}$

Table 3

Absolute Deviations from True Limit Deviations of Z Transformation

Method of Transformation	Absolute Deviation from "True" Upper Limit	Absolute Deviation from "True" Lower Limit
Soffer	$Dev\hat{\theta}_{1UCL} = \left\|\hat{\theta}_{1UCL} - \theta_{UCL}\right\|$	$Dev\hat{\theta}_{1LCL} = \left\|\hat{\theta}_{1LCL} - \theta_{LCL}\right\|$
Rocke	$Dev\hat{\theta}_{2UCL} = \left\|\hat{\theta}_{2UCL} - \theta_{UCL}\right\|$	$Dev\hat{\theta}_{2LCL} = \left\|\hat{\theta}_{2LCL} - \theta_{LCL}\right\|$
Chan/Xiao	$Dev\hat{\theta}_{3UCL} = \left\|\hat{\theta}_{3UCL} - \theta_{UCL}\right\|$	$Dev\hat{\theta}_{3LCL} = \left\|\hat{\theta}_{3LCL} - \theta_{LCL}\right\|$

Table 4

u-Chart Comparison

The conditions for the simulation study used a variable sample size which was uniformly distributed. The three sample size ranges used in the simulation where (10, 100), (100, 500), and (500, 1500). The number of "defects" per unit was produced using c as a Poisson random variable and u as the average nonconformances per unit inspected. Under this study, c was randomly generated at averages of 1.0, 1.5, 2.0, 2.5, and 3.0. An initial simulation run of 200 trials was used to establish process averages on which the control limits would be based. The main simulation consisted of 10,000 trials for each of the combination of ranges of c and of n. Table 5 shows the deviation from upper and lower limits, while Table 6 provides deviation from the "true" estimator values calculations. To make the suitable comparison, the deviations in Table 8 are divided by the standard error of each transformation, thus comparisons are made in terms of standard deviation.

Deviations from Limits

Method of Transformation	Deviation from Upper Limit	Deviation from Lower Limit
Duncan Z	$\theta_{UCL} = UCL_Z - Z$	$\theta_{LCL} = Z - LCL_Z$
Soffer	$\hat{\theta}_{1UCL} = UCL_{Soffer} - u_{Soffer}$	$\hat{\theta}_{1LCL} = u_{Soffer} - LCL_{Soffer}$
Rocke	$\hat{\theta}_{2UCL} = UCL_{Rocke} - u_{Rocke}$	$\hat{\theta}_{2LCL} = u_{Rocke} - LCL_{Rocke}$
Chan/Xiao	$\hat{\theta}_{3UCL} = UCL_{Chan/Xiao} - u_{Chan/Xiao}$	$\hat{\theta}_{3LCL} = u_{Chan/Xiao} - LCL_{Chan/Xiao}$

Table 5

Absolute Deviations from True Limit Deviations of Z Transformation

Method of Transformation	Absolute Deviation from "True" Upper Limit	Absolute Deviation from "True" Lower Limit
Soffer	$Dev\hat{\theta}_{1UCL} = \left\lvert \hat{\theta}_{1UCL} - \theta_{UCL} \right\rvert$	$Dev\hat{\theta}_{1LCL} = \left\lvert \hat{\theta}_{1LCL} - \theta_{LCL} \right\rvert$
Rocke	$Dev\hat{\theta}_{2UCL} = \left\lvert \hat{\theta}_{2UCL} - \theta_{UCL} \right\rvert$	$Dev\hat{\theta}_{2LCL} = \left\lvert \hat{\theta}_{2LCL} - \theta_{LCL} \right\rvert$
Chan/Xiao	$Dev\hat{\theta}_{3UCL} = \left\lvert \hat{\theta}_{3UCL} - \theta_{UCL} \right\rvert$	$Dev\hat{\theta}_{3LCL} = \left\lvert \hat{\theta}_{3LCL} - \theta_{LCL} \right\rvert$

Table 6

The Results

Results for the p-chart comparisons are summarized in table 7, giving the pair-wise comparisons for each of the calculated comparisons.

p-Chart Pair-wise Comparison Summary

	p=.01	p=.05	p=.10	p=.15	p=.20
n=U(10,100)	UCL use Chan/Xiao or Rocke LCL use Soffer	UCL use Rocke or Chan/Xiao LCL use Chan/Xiao	UCL use Rocke or Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao or Rocke LCL use Chan/Xiao	UCL use Chan/Xiao or Rocke LCL use Chan/Xiao
n=U(100,500)	UCL use Chan/Xiao or Rocke LCL use Chan/Xiao or Rocke	UCL use Soffer LCL use Chan/Xiao or Rocke	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Soffer LCL use Chan/Xiao	UCL use any LCL use any
n=U(500,1750)	UCL use Soffer LCL use Chan/Xiao or Rocke	UCL use Soffer LCL use Chan/Xiao	UCL use Soffer LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao

Table 7

As previously mentioned, this table reflects the "pure" comparisons as calculated by the computer and does not take into account the concern of relevant difference. The better Pitman performer under the simulated conditions is given for each combination of conditions. In cases where the probability difference between two transformations was less than 5%, the higher probability transformation is given, with the other offered as an alternative.

As seen in Table 7, appropriateness of the transformation method does depend on the conditions of the process, as well as the importance of monitoring the process more effectively at the upper or lower control limit. When the process has a low proportion nonconforming and a relatively small variable sample size, the practitioner should employ the Soffer transformation when interested in monitoring against performance to the lower limit, such as cases of looking to identify improvements in the process. Under these same conditions, the practitioner should utilize the Chan/Xiao transformation when concerned with the stability of the process toward the upper limit when monitoring for process degradation.

Of particular interest in this finding is that under conditions of higher proportion nonconforming (p=0.20), the Chan/Xiao transformation was preferable to the others under all cases of sample size. This would direct the practitioner with higher proportion nonconforming rates to use the Chan/Xiao transformation in all cases. In fact, when using the Pitman Criterion under the simulated conditions, the Chan/Xiao transformation is the choice transformation in eight of the 15 sets of conditions for the upper control limit, and 14 of the 15 sets of conditions for the lower control limit.

The results of the u-chart simulation are summarized in Table 8, which displays the results for each of the process configurations studied.

u-Chart Pair-wise Comparison Summary

	c=1.0	c=1.5	c=2.0	c=2.5	c=3.0
n=U (10,100)	UCL use Soffer LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao
n=U (100,500)	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Rocke or Soffer LCL use Rocke or Soffer	UCL use Rocke or Soffer LCL use Rocke or Soffer	UCL use Rocke or Soffer LCL use Rocke or Soffer
n=U (500,1500)	UCL use Soffer LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Chan/Xiao LCL use Chan/Xiao	UCL use Rocke LCL use Rocke

Table 8

When the simulation calculated that one transformation performs better than the others, that transformation is listed. If two transformations are similar in their probability of nearness (within 5%), the transformation which calculated to be nearer is provided, with the other transformation offered as an alternate. As an example, when c=1.0 and the sample size ranges from 10 to 100, the appropriate transformation when concerned with the UCL would be the Soffer based transformation, while the Chan/Xiao transformation would be better suited when concerned with the LCL.

An Interesting Finding

A quick look at the summary results would indicate that the Chan/Xiao transformation outperformed the others under more sets of process conditions. This observation would lead the practitioner to the conclusion that if consistency of charting is important across the many processes within an operation, using the Chan/Xiao transformation would be most suited as a facility-wide standard. But upon closer inspection of the data generated in this simulation, something a little more unexpected was discovered.

While the computer simulation was set up to make the comparisons automatically, no level of relevance was programmed into the simulation. This left the computer to make comparisons of numbers that were calculated and carried to far more decimal places than are relevant in most studies and certainly beyond the relevance of most process applications. A second simulation was run, this time rounding the standardized deviations from control limits to the fourth decimal place. In application, it seems reasonable that the fourth decimal place should be sufficient for relevance. In a closer review of the data, the differences when present were actually found at the 15th decimal place. These results are a reminder that when using computers to perform comparison calculations, a review of the raw data is always a good idea.

Table 9 provides the results of the comparison for the p-chart transformations after rounding the absolute deviations to the fourth decimal place.

p-Chart Pair-wise Comparison Summary after Rounding

	p=.01	p=.05	p=.10	p=.15	p=.20
n=U(10,100)	UCL use Chan/Xiao or Rocke LCL use Any	UCL use Rocke or Chan/Xiao LCL use Any	UCL use Rocke or Chan/Xiao LCL use Any	UCL use Chan/Xiao or Rocke LCL use Any	UCL use Any LCL use Any
n=U(100,500)	UCL use Any LCL use Chan/Xiao or Rocke	UCL use Any LCL use Chan/Xiao or Rocke	UCL use Any LCL use Chan/Xiao	UCL use Any LCL use Chan/Xiao	UCL use Any LCL use any
n=U(500,1750)	UCL use Any LCL use Chan/Xiao or Rocke	UCL use Any LCL use Any	UCL use Any LCL use Any	UCL use Any LCL use Any	UCL use Chan/Xiao LCL use Any

Table 9

These "relevant" differences are compared using the Pitman Criterion and show that from a relevant or practical sense, only 10 of the 30 sets of conditions have relevant closer/nearer estimators. At the smaller sample sizes, deviation from the UCL is better estimated using either the Rocke or Chan/Xiao transformations. Soffer (1981) pointed out that under conditions of small sample sizes, the transformation had potential drawbacks, which the simulation and comparison identify. From a more general sense, when taking the relevant decimal places into account, the use of either the Rocke transformation or the Chan/Xiao transformation are more suitable under the studied conditions. Soffer points this out as a situational issue of the transformation.

When only concerning ourselves with calculations up to the 4^{th} decimal place, there were no differences between any of the u-chart transformations. In all cases, the differences between the standardized deviations of transformations from their control limits and the deviation of the standardized u were all beyond the 4^{th} decimal place. Although the transformations are all different, though similar, these differences do not change the relevant performance of any of the transformations.

Detection Errors

Another practical measure was to identify cases where the transformation under consideration plotted either a "false alarm," plotting a point beyond the control limits when it should not have, or "no alarm," not plotting a point beyond the control limits when it should have. False alarms are an obvious concern to the practitioner because they lead to investigations and evaluations of the process when no special cause variation is actually detected. On the other hand, "no alarm" means that the process does exhibit special cause variation and the control chart does not trigger an investigation. Table 10 shows the conditions under which each of the p-chart transformations plotted one of these false/no alarms. (u-chart detection errors were not simulated since the initial findings did not note any differences in the methods of transformation).

Detection Error Rates

Transformation and Conditions	Number of Plotting Errors	Percent Plotted Errors
Soffer FA at LCL when p=.15 and n varies between (10,100)	1	0.01
Rocke FA LCL when p=.15 and n varies between (10,100)	1	0.01
Chan FA LCL when p=.15 and n varies between (10,100)	1	0.01
Soffer NA UCL when p=.15 and n varies between (10,100)	5	0.05
Soffer NA UCL when p=.20 and n varies between (10,100)	14	0.14
Soffer NA UCL when p=.10 and n varies between (10,100)	28	0.28
Soffer NA UCL when p=.05 and n varies between (10,100)	76	0.76
Soffer NA UCL when p=.01 and n varies between (10,100)	159	1.59
Soffer FA LCL when p=.01 and n varies between (500,1750)	1656	16.56

FA = False alarm NA = No Alarm

Table 10

As shown in Table 10, all three transformations had one false alarm in 10,000 trials when p was set at 0.15 and n varies. The Soffer transformation also had issues of false and no alarms under other sets of conditions. Particular concern would be the false alarm rate when p is small and the size of the sample is larger. This seems to be largely due to conditions where the sample size requires using the alternative LCL when the transformed value is negative.

Conclusion

As we would tend to expect when trying to apply theoretical methods to real problems, no one transformation method is better than the others in all cases. Practitioners wanting to use the p-chart with variable sample sizes must evaluate the conditions under which their specific process exists, and make the appropriate selection of methods of transformation. From a more practical and relevant standpoint, the transformations proposed by Rocke and Chan/Xiao tend to outperform the Soffer transformation. If the practitioner has a number of processes to chart and wants to use one standard method of transformation for consistency across all processes, selection of either the Rocke transformation or the Chan/Xiao transformation should perform equally from both a practical and a relevant standpoint.

In situations with variable sample size, transformations used in plotting u-charts provide both the ease of interpretation against the straight line control charts and plotting points in a more readily recognizable unit of measure. This study has shown that while there are some calculated differences in the performance levels of each transformation under specific process conditions, there is no relevant difference that would be expected to appear in application. Although there may be cases where the practitioner is concerned with differences beyond the 4^{th} decimal place, a simple review of the data can show where these differences may occur, and the practitioner is free to make decisions based on that outcome.

In cases where both p-charts and u-charts are used with varying sample sizes, the practitioner would likely want to use the transformation most suited for their p-chart processes, and use the same type transformation for u-charts. This will provide consistency throughout the facility and provide the best method of plotting these attribute control charts in cases of varying sample sizes.

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